SERIES TEMPORALES

Escribimos los siguientes modelos:

AR(1): $y_t = \emptyset y_{t-1} + \varepsilon t$

AR(2): $y_t = \emptyset_1 y_{t-1} + \emptyset_2 y_{t-2} + \varepsilon_t$

MA(1): $y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1}$

MA(2): $y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$

ARMA(1,1): $y_t = \emptyset y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$

ARMA(2,2): $y_t = \emptyset_1 y_{t-1} + \emptyset_2 y_{t-2} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$

Propiedades de las series estacionarias:

$E(y_t) = \mu$, para todo t $\quad$ $VAR(y_t) = VAR(y_{t-1})$ para todo t

$E(\varepsilon_t) = 0$, para todo t $\quad$ $VAR(\varepsilon_t) = \sigma^2$ para todo t

Calculo de valores estadísticos.

AR(1)

Media

$\mu = E(y_t) = E(\emptyset y_{t-1} + \varepsilon_t) = \emptyset E(y_{t-1}) + E(\varepsilon_t) = \emptyset \mu + 0 =$

$\mu = \emptyset \mu \quad \mu = 0$

Varianza

$$\text{VAR}(y_t) = \text{VAR}(\emptyset y_{t-1} + \varepsilon_t) = \emptyset^2 \text{ VAR}(y_{t-1}) + \text{VAR}(\varepsilon_t) = \emptyset^2 \text{VAR}(y_t) + \sigma^2$$

$$\text{VAR}(y_t) - \emptyset^2 \text{ VAR}(y_{t-1}) = \sigma^2$$

$$(1-\emptyset^2) \text{ VAR}(y_t) = \sigma^2$$

$$\text{VAR}(y_t) = \frac{\sigma^2}{1-\emptyset^2} \quad ; \quad V_0 = \frac{\sigma^2}{1-\emptyset^2}$$

Autocovarianzas

$$V_1 = \text{COV}(y_t, y_{t+1}) = \text{COV}(y_t, \emptyset y_t + \varepsilon_{t+1}) = \emptyset \text{ COV}(y_t, y_t) + \text{COV}(y_t, \varepsilon_{t+1}) = \emptyset V_0 = \emptyset \frac{\sigma^2}{1-\emptyset^2}$$

$$V_2 = \text{COV}(y_t, y_{t+2}) = \text{COV}(y_t, \emptyset y_{t+1} + \varepsilon_{t+2}) = \text{COV}(y_t, \emptyset y_{t+1}) + \text{COV}(y_t, \varepsilon_{t+2}) = \emptyset V_1 = \emptyset^2 V_0$$

$$V_k = \emptyset^k V_0$$

Autocorrelaciónes

$$\rho_0 = \frac{V_0}{V_0} = 1$$

$$\rho_1 = \frac{V_1}{V_0} = \frac{\emptyset V_0}{V_0} = \emptyset$$

$$\rho_1 = \frac{V_2}{V_0} = \frac{\emptyset^2 V_0}{V_0} = \emptyset^2$$

$$\rho_k = \frac{V_k}{V_0} = \emptyset^k$$

AR(2)

Media

$\mu = E(y_t) = E(\emptyset_1 y_{t-1} + \emptyset_2 y_{t-2} + \varepsilon_t) =$

$\emptyset_1 E(y_{t-1}) + \emptyset_2 E(y_{t-2}) + E(\varepsilon t) = \emptyset_1 \mu + \emptyset_2 \mu + 0$

$\mu = (\emptyset_1 + \emptyset_2)\mu \quad \mu = 0$

Varianza

$V_0 = VAR(y_t) = VAR(\emptyset_1 y_{t-1} + \emptyset_2 y_{t-2} + \varepsilon_t) = \emptyset_1^2 VAR(y_{t-1}) +$

$\emptyset_2^2 VAR(y_{t-2}) + VAR(\varepsilon_t) + 2\emptyset_1^2\emptyset_2^2 Cov(y_{t-1}, y_{t-2})$
$= \emptyset_1^2 V_0 + \emptyset_2^2 V_0 + 2\emptyset_1^2\emptyset_2^2 V_1 + \sigma^2$

$(1 - \emptyset_1^2 - \emptyset_2^2)\ V_0 = \sigma^2 + 2\emptyset_1^2\emptyset_2^2 V_1 ;\ V_0 = \frac{\sigma^2 + 2\emptyset_1^2\emptyset_2^2 V_1}{1 - \emptyset_1^2 - \emptyset_2^2}$

Autocovarianzas

$V_1 = COV(y_t, y_{t+1}) = COV(y_t, \emptyset_1 y_t + \emptyset_2 y_{t-1} + \varepsilon_{t+1}) =$

$\emptyset_1 COV(y_t, y_t) + \emptyset_2 COV(y_t, y_{t-1}) + COV(y_t, \varepsilon_{t+1}) =$

$\emptyset_1 V_0 + \emptyset_2 V_1 + 0$

$V_1 = \emptyset_1 V_0 + \emptyset_2 V_1$

$(1 - \emptyset_2) V_1 = \emptyset_1 V_0$

$$V_1 = \frac{\emptyset_1 V_0}{1 - \emptyset_2}$$

$V_2 = \mathrm{COV}(y_t, y_{t+2}) = \mathrm{COV}(y_t, \emptyset_1 y_{t+1} + \emptyset_2 y_t + \varepsilon_{t+2})$

$= \emptyset_1 \mathrm{COV}(y_t, y_{t+1}) + \emptyset_2 \mathrm{COV}(y_t, y_t)$

$+\mathrm{COV}(y_t, \varepsilon_{t+2}) = \emptyset_1 V_1 + \emptyset_2 V_0 + 0$

$V_3 = \mathrm{COV}(y_t, y_{t+3}) = \mathrm{COV}(y_t, \emptyset_1 y_{t+2} + \emptyset_2 y_{t+1} + \varepsilon_{t+3})$

$= \emptyset_1 \mathrm{COV}(y_t, y_{t+2}) + \emptyset_2 \mathrm{COV}(y_t, y_{t+1})$

$+\mathrm{COV}(y_t, \varepsilon_{t+3}) = \emptyset_1 V_2 + \emptyset_2 V_1$

$V_4 = \mathrm{COV}(y_t, y_{t+4}) = \mathrm{COV}(y_t, \emptyset_1 y_{t+3} + \emptyset_2 y_{t+2} + \varepsilon_{t+4})$

$= \emptyset_1 \mathrm{COV}(y_t, y_{t+3}) + \emptyset_2 \mathrm{COV}(y_t, y_{t+2})$

$+\mathrm{COV}(y_t, \varepsilon_{t+4}) = \emptyset_1 V_3 + \emptyset_2 V_2$

$V_k = \emptyset_1 V_{k-1} \mathrm{V3} + \emptyset_2 V_{k-2} \quad \text{para todo k} > 1$

<u>Autocorrelaciones</u>

$$\rho_0 = \frac{V_0}{V_0} = 1$$

$$\rho = \frac{V_1}{V_0}$$

$$\rho_2 = \frac{\emptyset_1 V_1 + \emptyset_2 V_0}{V_0}$$

$$\rho_3 = \frac{\emptyset_1 V_2 + \emptyset_2 V_1}{V_0}$$

$$\rho_k = \frac{\emptyset_1 V_{k-1} + \emptyset_2 V_{k-2}}{V_0}$$

MA(1)

Media

$$\mu = \mathrm{E}(\mathrm{y}_t) = \mathrm{E}(\varepsilon_t + \theta\varepsilon_{t-1}) = \mathrm{E}(\varepsilon_t) + \theta\mathrm{E}(\varepsilon_{t-1}) = 0$$

Varianza

$$\mathrm{VAR}(\mathrm{y}_t) = \mathrm{VAR}(\varepsilon_t + \theta\varepsilon_{t-1}) = \mathrm{VAR}(\varepsilon_t) + \theta^2\mathrm{VAR}(\varepsilon_{t-1}) =$$

$$\sigma^2 + \theta^2\,\sigma^2 = V_0 = (1+\theta^2)\,\sigma^2$$

Autocovarianzas

$V_1 = \mathrm{COV}(\mathrm{y}_t, \mathrm{y}_{t+1}) = \mathrm{COV}(\varepsilon_t + \theta\varepsilon_{t-1}, \varepsilon_{t+1} + \theta\varepsilon_t) = \theta cov(\varepsilon_t, \varepsilon_t) = \theta\sigma^2$

$V_2 = \mathrm{COV}(\mathrm{y}_t, \mathrm{y}_{t+2}) = \mathrm{COV}(\varepsilon_t + \theta\varepsilon_{t-1}, \varepsilon_{t+2} + \theta\varepsilon_t + 1) = 0$

$V_k = 0$ Para todo valor de $\mathrm{k} > 1$

Autocorrelaciones

$$\rho_0 = \frac{V_0}{V_0} = 1$$

$$\rho_1 = \frac{V_1}{V_0} = \frac{\theta\sigma^2}{(1+\theta^2)\,\sigma^2} = \frac{\theta}{1+\theta^2}$$

$$\rho_2 = \frac{V_2}{V_0} = 0$$

$\rho_k \ = 0$ para todo $k > 1$

MA(2)

Media

$\mu = \mathrm{E}(\mathrm{y}_t) = \mathrm{E}(\varepsilon_t + \theta\varepsilon_{t-1} + \theta_2\varepsilon_{t-2})$

Varianza

$\mathrm{VAR}(\mathrm{y}_t) = \mathrm{VAR}(\varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}) =$

$\mathrm{VAR}(\varepsilon_t) + \theta_1^2\mathrm{VAR}(\varepsilon_{t-1}) + \theta_1^2\mathrm{VAR}(\varepsilon_{t-2}) =$

$\sigma^2 + \theta_1^2\,\sigma^2 + \theta_2^2\sigma^2 = (1 + \theta_1^2 + \theta_2^2)\sigma^2$

Autocovarianzas

$V_1 = \mathrm{COV}(\mathrm{y}_t, \mathrm{y}_{t+1}) = \mathrm{COV}(\varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}, \varepsilon_{t+1} + \theta_1\varepsilon_t + \theta_2\varepsilon_{t-1})$

$= \theta_1 cov(\varepsilon_t, \varepsilon_t) + \theta_1\theta_2 ov(\varepsilon_{t-1}, \varepsilon_{t-1}) = \theta_1\sigma^2 + \theta_1\theta_2\sigma^2$

$= (\theta_1 + \theta_1\theta_2)\,\sigma^2$

$V_2 = \mathrm{COV}(\mathrm{y}_t, \mathrm{y}_{t+2}) = \mathrm{COV}(\varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}, \varepsilon_{t+2} + \theta_1\varepsilon_{t+1} + \theta_2\varepsilon_t)$

$= \theta_2 ov(\varepsilon_t, \varepsilon_t) = \theta_2\sigma^2$

$V_3 = \mathrm{COV}(\mathrm{y}_t, \mathrm{y}_{t+3}) =$

$\mathrm{COV}(\varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}, \varepsilon_{t+3} + \theta_1\varepsilon_{t+2} + \theta_2\varepsilon_{t+1}) = 0$

$V_k \ = 0$ Para todo valor de $k > 2$

Autocorrelaciones

$$\rho_0 = \frac{V_0}{V_0} = 1$$

$$\rho_1 = \frac{V_1}{V_0} = \frac{(\theta_1 + \theta_1\theta_2)\,\sigma^2}{(1 + \theta_1^2 + \theta_2^2)\,\sigma^2} = \frac{\theta_1 + \theta_1\theta_2}{1 + \theta_1^2 + \theta_2^2}$$

$$\rho_2 = \frac{V_2}{V_0} = \frac{\theta_2\sigma^2}{(1 + \theta_1^2 + \theta_2^2)\,\sigma^2} = \frac{\theta_2}{1 + \theta_1^2 + \theta_2^2}$$

$$\rho_3 = 0$$

Condiciones estacionaridad

MA - Siempre es estacionario

AR - Si las raíces, en valor absoluto, son menores que 1

ARMA - Si la parte AR es estacionaria

De manera concreta:

AR(1) $|\emptyset| < 1$

AR(2) $\emptyset_1 + \emptyset_2 < 1$

$\emptyset_2 - \emptyset_1 < 1$

$\emptyset_2 > -1$

Para valores mayores de 2 es necesario calcular las raíces con un ordenador.

Selección de modelos

Existen tres opciones para determinar cuál es el mejor modelo

- $A\dot{I}C = -2\log - \text{lik} + 2\text{k}$ $\quad$ $\text{k} =$ números de parámetros
- $BIC = -2\log - \text{lik} + \text{k} * \ln(\text{n})$ n = números de observaciones

  En ambos el mejor es el menor.

- $LM = 2(\log - \text{Lik}_{MNR} - \text{Log} - \text{lik}_{MR}) \sim x_k^2$
  siendo k el numero d restricciones

Se compara con el valor en tablas de la x^2

Tienen que estar anidados es decir ser uno modificación de otro.

No se puede sustraer de ambas partes a la vez (AR y MA)

Ejemplo:

Elegir el mejor modelo N=1000

Modelo	Log-lik
AR (2)	50,7
MA (2)	29,3
ARMA(1,1)	50,2
ARMA(2,2)	55,3

AIC

$AR(2) = -2 * 50{,}7 + 2 * 2 = -97{,}4$

$MA(2) = -2 * 29{,}3 + 2 * 2 = -54{,}6$

$ARMA(1{,}1) = -2 * 50{,}2 + 2 * 2 = -96{,}4$

$ARMA(2{,}2) = -2 * 55{,}3 + 2 * 4 = -102{,}4$

BIC

$AR(2) \ = -2 * 50{,}7 + 2\ln(1000) = \ -87{,}58$

$MA(2) \ = \ -2 * 29{,}3 + 2\ln(1000) = \ -44{,}78$

$ARMA(1{,}1) = \ -2 * 50{,}2 + 2\ln(1000) = \ -86{,}58$

$ARMA(2{,}2) = \ -2 * 55{,}3 + 2\ln(1000) = \ -82{,}77$

LM

$AR(2)$ versus MA(2) (NO)

$AR(2)$ versus ARMA(1,1) (No)

$AR(2)$ versus ARMA(2,2) (SI)

$H_0: \theta_1 = \theta_2 = 0$

H_1: Algún $\theta_i \neq 0$

$LM \ = 2 * (55{,}3 - 50{,}7) = 9 \sim x_2^2 \ = 5{,}99$

Rechazo HO elijo ARMA (2,2)

$MA(2)$ versus ARMA(1,1) (NO)

$MA(2)$ versus ARMA(2,2) (SI)

$H_0: \emptyset_1 = \emptyset_2 = 0$

H_1: Algún $\emptyset_i$ ≠0

$LM = 2 * (55{,}2 - 29{,}3) = 51{,}8 \sim x_2^2 =$

5,99 rechazo HO elijo ARMA (2,2)

$ARMA(1{,}1)$ versus ARMA(2,2) (NO)

Respuesta final ↔ ARMA (2,2)

Predicciones

ARMA(2,2)

$$E(Y_{T+k|T}) = E(\emptyset_1 Y_{T+k-1|T} + \emptyset_2 Y_{T+k-2|T} + \varepsilon_{t+K|T} + \theta_1 \varepsilon_{t+K-1|T} + \theta_2 \varepsilon_{t+K-2|T})$$

$$\tilde{y}_{T+k|T} = \emptyset_1 \tilde{y}_{T+k-1|T} + \emptyset_2 \tilde{y}_{T+k-2|T} + \tilde{\varepsilon}_{T+k|T} + \theta_1 \tilde{\varepsilon}_{T+k-1|T} + \theta_2 \tilde{\varepsilon}_{T+k-2|T}$$

<u>K=1</u>

$$\tilde{y}_{T+1|T} = \emptyset_1 \tilde{y}_{T|T} + \emptyset_2 \tilde{y}_{T-1|T} + \tilde{\varepsilon}_{T+1|T} + \theta_1 \tilde{\varepsilon}_{T|T} + \theta_2 \tilde{\varepsilon}_{T-1|T}$$

$$\tilde{y}_{T+1|T} = \emptyset_1 y_t + \emptyset_2 y_{t-1} + \theta_1 \varepsilon_{t_t} + \theta_2 \varepsilon_{t_{t-1}}$$

<u>K=2</u>

$$\tilde{y}_{T+2|T} = \emptyset_1 \tilde{y}_{T+1|T} + \emptyset_2 \tilde{y}_{T|T} + \tilde{\varepsilon}_{T+2|T} + \theta_1 \tilde{\varepsilon}_{T+1|T} + \theta_2 \tilde{\varepsilon}_{T|T}$$

$$\tilde{y}_{T+2|T} = \emptyset_1 \tilde{y}_{T+1|T} + \emptyset_2 y_t + \theta_2 \varepsilon_t$$

$$\tilde{y}_{T+2|T} = \emptyset_1(\emptyset_1 y_t + \emptyset_2 y_{t-1} + \theta_1 \varepsilon_t + \theta_2 \varepsilon_{t-1}) + \emptyset_2 y_t + \theta_2 \varepsilon_t$$

K=3

$$\tilde{y}_{T+3|T} = \emptyset_1 \tilde{y}_{T+2|T} + \emptyset_2 \tilde{y}_{T+1|T} + \tilde{\varepsilon}_{T+3|T} + \theta_1 \tilde{\varepsilon}_{T+2|T} + \theta_2 \tilde{\varepsilon}_{T+1|T}$$

$$\tilde{y}_{T+3|T} = \emptyset_1 \tilde{y}_{T+2|T} + \emptyset_2 \tilde{y}_{T+1|T}$$

$$\tilde{y}_{T+3|T} = \emptyset_1^2(\emptyset_1 y_t + \emptyset_2 y_{t-1} + \theta_1 \varepsilon_t + \theta_2 \varepsilon_{t-1}) + \emptyset_1(\emptyset_2 y_t + \theta_2 \varepsilon_t) +$$

$$\emptyset_2(\emptyset_1 y_t + \emptyset_2 y_{t-1} + \theta_1 \varepsilon_t + \theta_2 \varepsilon_{t-1})$$

Error Cuadrático Medio

$$ECM = \mathrm{E}(\tilde{y}_{T+k|T} - y_T)^2$$

K=1

$$ECM_1 = \mathrm{E}(\emptyset_1 y_t + \emptyset_2 y_{t-1} + \theta_1 \varepsilon_t + \theta_2 \varepsilon_{t-1})$$

$$-(\emptyset_1 y_t + \emptyset_2 y_{t-1} + \varepsilon_{t+1} + \theta_1 \varepsilon_t + \theta_2 \varepsilon_{t-2})^2$$

$$= \mathrm{E}(\varepsilon_t)^2 = \sigma^2$$

K=2

$$ECM_2 = \mathrm{E}(\emptyset_1(\emptyset_1 y_t + \emptyset_2 y_{t-1} + \theta_1 \varepsilon_t + \theta_2 \varepsilon_{t-1}) + \emptyset_2 y_t + \theta_2 \varepsilon_t$$

$$-(\emptyset_1 y_{t+1} + \emptyset_2 y_t + \varepsilon_{t+2} + \theta_1 \varepsilon_{t+1} + \theta_2 \varepsilon_t))^2 =$$

$$= \mathrm{E}(\emptyset_1(\emptyset_1 y_t + \emptyset_2 y_{t-1} + \theta_1 \varepsilon_t + \theta_2 \varepsilon_{t-1}) - (\emptyset_1 y_{t+1} + \varepsilon_{t+2} + \theta_1 \varepsilon_{t+1})^2$$

K=3

$$ECM_3 = E(\emptyset_1^2(\emptyset_1 \mathrm{y_t} + \emptyset_2 \mathrm{y_{t-1}} + \theta_1\, \varepsilon_\mathrm{t} + \theta_2\, \varepsilon_{\mathrm{t-1}}\,) + \emptyset_1(\emptyset_2 \mathrm{y_t} + \theta_2\, \varepsilon_\mathrm{t}) + \emptyset_2(\emptyset_1 \mathrm{y_t} + \emptyset_2 \mathrm{y_{t-1}} + \theta_1\, \varepsilon_\mathrm{t} + \theta_2\, \varepsilon_{\mathrm{t-1}}\,) - (\emptyset_1 y_{t+2} + \emptyset_2 y_{t+1} + \varepsilon_{t+3} + \theta_1 \varepsilon_{t+3} + \theta_2 \varepsilon_{t+1})$$

$$ECM_3 = E(\emptyset_1^2(\emptyset_1 \mathrm{y_t} + \emptyset_2 \mathrm{y_{t-1}} + \theta_1\, \varepsilon_\mathrm{t} + \theta_2\, \varepsilon_{\mathrm{t-1}}\,) + \emptyset_1(\emptyset_2 \mathrm{y_t} + \theta_2\, \varepsilon_\mathrm{t}) + \emptyset_2(\emptyset_1 \mathrm{y_t} + \emptyset_2 \mathrm{y_{t-1}} + \theta_1\, \varepsilon_\mathrm{t} + \theta_2\, \varepsilon_{\mathrm{t-1}}\,) - (\emptyset_1 y_{t+2} + \emptyset_2 y_{t+1} + \varepsilon_{t+3} + \theta_1 \varepsilon_{t+3} + \theta_2 \varepsilon_{t+1}))^2$$